Ordinary differential equations

Assignment 3

(Green's functions)

August–November Semester 2024

Department of Mathematics, Indian Institute of Technology Palakkad

11 October, 2024 (Friday) *Due: 21 October, 2024 (11:50 PM) (Monday) In Moodle.*

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1. Find the Green's functions for the following boundary value problems.

   (a) $L(y) = (1-x)^2 y'' - 2xy = 0$ with the boundary conditions $y(0) = 0 = y'(1)$.

   (b) $L(y) = y'' + a^2 y = 0$, where a is a constant and $y(0) = 0 = y(1)$.

2. Find the conditions under which the boundary value problem $y''(x) = f(x)$ with conditions $y(0) = 0$ and $y(1) - y'(1) = 1$ has a solution. Solve this boundary value problem using the method of Green's function, when it admits a solution.

3. Determine the Green's function for the boundary value problem

$$xy'' + y' = -f$$

   under the conditions $y(1) = 0$ and $\lim_{x\to 0} |y(x)| < \infty$.

4. Consider the nonhomogeneous boundary value problem with continuous coefficients

$$\begin{aligned} L(y) &= y'' + p(x)y' + q(x)y = g(x) \\ U_1(y) &= a_1 y(a) + a_2 y'(a) + b_1 y(b) + b_2 y'(b) = \alpha \\ U_2(y) &= a_3 y(a) + a_4 y'(a) + b_3 y(b) + b_4 y'(b) = \beta. \end{aligned}$$

   If the associated homogeneous boundary value problem

$$L(y) = 0 \quad \text{and} \quad U_j(y) = 0$$

   has a trivial solution only, prove that the non-homogeneous problem has a unique solution.

5. Determine the generalised Green's function for the boundary value problem

$$((1-x^2)y')' = -f(x)$$

   under the conditions $\lim_{x\to\pm 1} |y(x)| < \infty$.