Ordinary differential equations

Assignment 2

(Existence theorems, maximal interval of existence, Wronskain, variation of parameters)

August–November Semester 2024

Department of Mathematics, Indian Institute of Technology Palakkad

12 September, 2024 (Thursday) *Due: 23 September, 2024 (11:50 PM) (Monday) In Moodle.*

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1. Construct all possible solutions of the ordinary differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2y}{x}$$

on $\mathbb{R}$. Among these solutions, find a unique solution on the largest subinterval of $\mathbb{R}$.

2. Consider the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 4x^{3/4}.$$

For a very small real number $\varepsilon > 0$, let φ_ε and φ denote the solutions for the above equation with the initial conditions $\varphi_\varepsilon(0) = 0$ and $\varphi(0) = \varepsilon$, respectively. Find these solutions such that $|\varphi(x) - \varphi_\varepsilon(x)| \to 0$ as $x \to 0$.

3. Let $y'(x) = |y|^{-3/4}y + x\sin(\pi/x)$ with the initial condition $x(0) = 0$. Show that the Cauchy-Peano approximation (piecewise polygonal approximation) need not converge as $\varepsilon \to 0$.

4. Use the Wronskian to prove that two solutions of the homogenenous equation $y'' + P(x)y' + Q(z)y = 0$ on an interval $[a, b]$ is linearly dependent if

   (a) they have a common zero x_0 in the interval

   (b) they have maxima or minima at the same point x_0 in the interval.

5. Find particular solutions of the following equations using the method of variation of parameters.

   (a) $y'' + y = \sec(x)$

   (b) $y'' + y = \cot^2(x)$

   (c) $y'' + y = \sec(x)\tan(x)$

   (d) $y'' + y = x\cos(x)$

   (e) $y'' + 2y' + 5y = \exp(-x)\sec(2x)$.