Ordinary differential equations

Assignment 1

(Introduction, Separation of Variables, Exact equations, Integrating factors)

August–November Semester 2024

Department of Mathematics, Indian Institute of Technology Palakkad

17 August, 2024 (Monday) *Due: 28 August, 2024 (11:50 AM) (Wednesday) In Moodle.*

Instructor. Dr Gopikrishnan Chirappurathu Remesan

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1. Construct an appropriate function F and a domain D to write the following differential equations in the general form.
   (a) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + \sin(x)\dfrac{\mathrm{d}y}{\mathrm{d}x} = \log(x)$.
   (b) $\exp\left(\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3} - \dfrac{\mathrm{d}y}{\mathrm{d}x}\right) + x\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)^2 = 0$.
   (c) $\displaystyle\sum_{j=0}^{n} a_j \left(\frac{\mathrm{d}^n y}{\mathrm{d}x^n}\right) = 0$ where $a_j : \mathbb{R} \to \mathbb{R}$ is a function for $0 \le j \le n$.

2. Show that the differential equation
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{|y|}, \quad y(0) = 0$$
has four different solutions through the point $(0, 0)$. Sketch these solutions in the (x, y) plane. Comment about the well–posedness of this problem.

3. Construct a continuous solution to the problem
$$\frac{\mathrm{d}y}{\mathrm{d}x} + p(x)y(x) = q(x)$$
where
$$p(x) = \begin{cases} \lambda_1 & \text{if } 0 \le x \le 1 \\ \lambda_2 & \text{if } x > 1 \end{cases} \quad \text{and} \quad q(x) = \begin{cases} \lambda_3 & \text{if } 0 \le x \le 1 \\ \lambda_4 & \text{if } x > 1. \end{cases}$$

4. Test for the exactness and solve the following differential equations
   (a) $(1/x - 1/y)\mathrm{d}x + (x/y^2)\mathrm{d}y = 0$
   (b) $(y\cos(x) - \sin(y))\mathrm{d}x + (\sin(x) - x\cos(y))\mathrm{d}y = 0$
   (c) $(1 - xy)^{-2}\mathrm{d}x + (y^2 + x^2(1 - xy)^{-2})\mathrm{d}y = 0$

5. A function is said to be periodic with period p is $f(x + np) = f(x)$, where n is an integer. Suppose that f is continuous and periodic with period p for all x. Show that if φ is a solution of the homogeneous equation
$$\frac{\mathrm{d}y}{\mathrm{d}x} + f(x)y(x) = 0$$
then $\varphi(x + p)$ is also a solution. Show that for some constant c, $\varphi(x + p) = c\varphi(x)$.