**Test 1**

MA1011A: Linear Algebra and Series                    Date: 05/09/2024 (Thursday)

Max. Marks: 20                                        Time: 08∶00 -08∶50 AM

**Instructions:**

(a) Cell phones and scientific calculators are **not** allowed within the exam hall.

(b) **No doubts will be clarified by the invigilators/course instructors.**

(c) Please state all the results which you use and justify your answers.

(1) Let $A$ and $B$ be $n \times n$ matrices. Suppose $AB = I_n$ (the identity matrix). In this problem, you will show $BA = I_n$ and thus $B = A^{-1}$. This is a multi-part problem and if you are unable to solve one part, you can still attempt the subsequent parts assuming the conclusion of prior parts.

   (a) (3 Marks) Show that if $C$ is another $n \times n$ matrix such that $CA = I_n$ then $B = C$.

   (b) (2 Marks) Find an $n \times n$ invertible matrix $P$ such that $PA$ is in RREF.

   (c) (1 Mark) Show that the bottom row of $P$ cannot be 0.

   (d) (3 Marks) Use the previous parts to conclude that $P = B$ and thus $A$ is invertible.

(2)  (a) (2 Marks) Let $V$ be a vector space and let $v, w \in V$. Show that if $v$ and $w$ are linearly independent then either $v = cw$ or $w = cv$ for some $c \in \mathbb{R}$.

   (b) (3 Marks) Fix a positive natural number $n$. Show that we can find $n$ vectors $v_1, \ldots, v_n \in \mathbb{R}^2$ such that any pair $v_i, v_j, i \neq j$ are linearly independent.

   (c) (3 Marks) Let $v_1, \ldots, v_k \in \mathbb{R}^m$ be such that $v_i \cdot v_j = 0$ (the dot product) whenever $i \neq j$. Show that $v_1, \ldots, v_k$ are linearly independent.

(3) (3 Marks) Using row operations, find the inverse of the matrix

$$\begin{bmatrix} 2 & 1 & 2 \\ 0 & 3 & -1 \\ 4 & 1 & 1 \end{bmatrix}.$$