Some properties of characteristic polynomial

Lemma. If A is a tolangular mulvix, then eigenvalues are the diagonal entoies

Pf. $\chi_{A}(\lambda) = (a_{11} - \lambda) - \cdots - (a_{nn} - \lambda)$ If A is topological are

Therefore roots are $\lambda_1 = a_{11}, \dots, \lambda = a_{nn}$

Observation. Suppose that $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigenvalues.

Then $\chi_{\mathcal{L}}(\lambda) = (\lambda - \lambda_1)(\lambda - \lambda_2) \cdots (\lambda - \lambda_h)$

 $= \lambda^{n} - (\lambda_{1} + \cdots + \lambda_{n}) \lambda^{n-1} + \cdots + (-1)^{n} \lambda_{1} \lambda_{2} \cdots \lambda_{n}$

Example: A = an an an

 $\chi_{A}(x) = del(A - \lambda I) = (a_{11} - \lambda I)(a_{22} - \lambda I) - a_{12}a_{21}$

 $= \lambda^{2} - \lambda (a_{11} + a_{22}) + a_{11} a_{22} - a_{12} a_{21}$ $= 4 \operatorname{vace}(\lambda) = \det(\lambda)$

 $= \lambda^2 - (\lambda_1 + \lambda_2) \lambda + \lambda_1 \lambda_2$

 $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ Example.

 $\chi_{A}(\lambda) = \det (A - \lambda I) = \det \begin{bmatrix} a_{11} - \lambda & a_{12} & a_{13} \\ a_{21} & a_{22} \lambda & a_{23} \end{bmatrix}$ $a_{31} \quad a_{32} \quad a_{32} \quad a_{33} \lambda$

 $= (a_{11} - \lambda) \left[(a_{22} - \lambda)(a_{32} - \lambda) - a_{23} a_{32} \right] + \cdots$

$$= (a_{11} - \lambda) (a_{22} - \lambda) (a_{33} - \lambda) - a_{23}a_{32}(a_{11} - \lambda) + \cdots$$

$$= -\lambda^{3} + (a_{11} + a_{22} + a_{33}) \lambda^{2} + \cdots + \cdots$$

In general the characteristic polynomial is of the from

$$\chi_{A}(x) = (-1)^{h} \left(\chi^{h} - (\log A) \chi^{h-1} + \cdots \right)$$

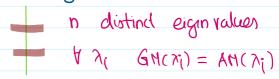
$$= (-1)^{h} \left(\chi^{h} - (\lambda_{1} + \cdots + \lambda_{n}) \chi^{h-1} + \cdots \right)$$
Comparison gives $\{ (A) = \lambda_{1} + \cdots + \lambda_{N} \}$

Lemma:
$$\lambda_1 \lambda_2 \cdots \lambda_h = \det(A)$$

Physical April A

DIAGONALIZATION

Suppose that Anxn how n lineably independent eigenvectors



Let the eigen rectors are vi, -- , un

$$\lambda v_j = \lambda_j v_j$$

Define a metroix P = [v1 v2 - vn]

$$AP = A \left[v_1 \quad v_2 \quad v_n \right]$$

$$= \left[Av_1 \quad Av_2 \quad Av_n \right]$$

$$= \left[\lambda_1 v_1 \quad \lambda_2 v_2 \quad \lambda_n v_n \right]$$

Let
$$D = diag(\lambda_1 - - \lambda_n)$$

$$AP = D[v_1 v_2 v_n]$$

$$= DP$$
Since $rank(P) = n$, P is in vertible. Therefore
$$D^{\dagger}AP = D$$
 (That is

$$\overline{D}^{\dagger}AP = D$$
 (That is A is similar to

 $D = \text{diag} (\lambda_1 \dots \lambda_n)$

If A matrix has n linearly independent eigenvectors, then I a medoix P = [v1 -- vn], vj are eigenvetors, and a diagonal med six $D = diag(\lambda_1 - \lambda_h)$ Such that $\hat{P}^{\dagger}AP = D$.

Dem: If A is similar to a diagonal matrix, we say A is dicegona lizable.

Thm. A is diagonalizable TA A has in linearly independent eign vertoss.

Pf. Suppose A is diagonalizable. Then
$$\exists P$$
 Such liked $\overline{P}^{\dagger}AP = D$

$$\Rightarrow$$
 AP = PD = DP

$$\Rightarrow$$
 $[AP - - AP^n] = [a_1P^1 - a_nP^n]$

Since P is an inv madrix, the Column are LI.